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A simple connection between neutrino oscillation and leptogenesis

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Abstract: The usual see-saw formula is modified by the presence of two Higgs triplets in left-right symmetric theories. The contribution from the left-handed Higgs triplet to the see-saw formula can dominate over the conventional one when the neutrino Dirac mass matrix is identified with the charged lepton or down quark mass matrix. In this case an analytic calculation of the lepton asymmetry, generated by the decay of the lightest right-handed Majorana neutrino, is possible. For typical parameters, the out-of-equilibrium condition for the decay is automatically fulfilled. The baryon asymmetry has the correct order of magnitude, as long as the lightest mass eigenstate is not much lighter then 10^{-6} to 10^{-8} eV, depending on the solution of the solar neutrino problem. A sizable signal in neutrinoless double beta decay can be expected, as long as the smallest mass eigenstate is not much lighter than 10^{-3} eV and the Dirac mass matrix is identified with the charged lepton mass matrix.

Keywords: Cosmology of Theories beyond the SM, Baryogenesis, Neutrino Physics, CP violation.

Contents

1. Introduction

The impressive evidence for non-vanishing neutrino masses [1] opens the possibility to study a number of new physics problems on a broader phenomenological basis. For the first time non Standard Model physics is seen. For example, the observed baryon asymmetry of the universe cannot be explained by conventional Standard Model physics [2]. Since the leptogenesis mechanism proposed by Fukugita and Yanagida [3] requires very massive right-handed neutrinos, it is a fruitful task to try to connect the properties of the known light neutrinos with their heavy right-handed counterparts. For this, certain assumptions concerning the high-energy part of the theory have to be made. Several recent articles dealt with this subject [4]–[12]. In [13] we argued that the left-handed Higgs triplet — which was absent in the previous papers — gives the most important contribution to the neutrino mass matrix (see also refs. [14]). In addition to the triplet contribution there is a correction from the usual see-saw [15] term $m_D\, M_R^{-1}\, m_D^T$. The large top quark mass makes this see-saw contribution comparable to the triplet contribution if m_D is identified with the up quark mass matrix. This is no longer the case if the mass scales in m_D are determined by the charged lepton or down quark masses. The neutrino mass matrix is then described mainly by the Higgs triplet contribution. This has the remarkable consequence that the mixing matrix, which diagonalizes the light left-handed neutrino mass matrix and is measured in oscillation experiments, is *identical* to the one which diagonalizes the heavy right-handed mass matrix, which governs the lepton asymmetry. In addition, the masses of the heavy right-handed Majorana neutrinos are proportional to the masses of the light left-handed ones. Apart from this aesthetically attractive property, the scenario allows to derive very simple analytical results on the baryon asymmetry in terms of the measured oscillation parameters and the yet unknown phases in the mixing matrix. We find that with natural choice of parameters the observed value of [16]

$$Y_B \simeq (0.1 \ldots 1) \cdot 10^{-10}, \tag{1.1}$$

is predicted. Within our scenario, it is possible to obtain a limit on the lightest mass eigenstate m_1, which is 10^{-6} (10^{-7}, 10^{-8}) eV for the small mixing (large mixing, vacuum) solution of the solar neutrino problem. Furthermore, the out-of-equilibrium condition $\Gamma_1 \lesssim H(M_1)$, where Γ_1 is the decay width of the lightest heavy Majorana and $H(M_1)$ the Hubble constant at the time of the decay, is also fulfilled. Finally, we make the connection to neutrinoless double beta decay and find that a sizable signal can be expected, as long as the smallest mass eigenstate is not much lighter than 10^{-3} eV and the Dirac mass matrix is identified with the charged lepton mass matrix.

The paper is organized as follows: in section 2 we summarize how leptogenesis and neutrino oscillations are connected in left-right symmetric theories. This section summarizes the framework described in our earlier paper [13]. In section 3 we perform an estimate of the baryon asymmetry, which explains almost all basic features found in the detailed numerical analysis, presented in section 4. The interesting connection to neutrinoless double beta decay is drawn in section 5. Our conclusions are presented in section 6.

2. Neutrino Oscillation and Leptogenesis in left-right symmetric theories

The light and heavy neutrino masses are obtained by diagonalizing

$$\begin{pmatrix} m_L & \tilde{m}_D \\ \tilde{m}_D^T & M_R \end{pmatrix}, \tag{2.1}$$

where m_L (M_R) is a left-handed (right-handed) Majorana and $\tilde{m}_D$ a Dirac mass matrix. This yields

$$m_\nu = m_L - \tilde{m}_D \, M_R^{-1} \, \tilde{m}_D^T \, . \tag{2.2}$$

The contribution of m_L is often neglected, even though it might play an important role in explaining the oscillation data [17]. This matrix is further diagonalized by

$$U_L^T\, m_\nu\, U_L = \mathrm{diag}(m_1, m_2, m_3)\,, \tag{2.3}$$

where m_i are the light neutrino masses. The entries of U_L are then measured in oscillation experiments, see section 3.1. The symmetric matrix M_R also appears in the Lagrangian

$$-\mathcal{L}_Y = \overline{l_{iL}}\, \frac{\Phi}{\langle \Phi \rangle}\, \tilde{m}_{Dij}\, N'_{Rj} + \frac{1}{2} \overline{N'^c_{Ri}}\, M_{Rij}\, N'_{Rj} + \text{ h.c.} \tag{2.4}$$

with l_{iL} the leptonic doublet and $\langle \Phi \rangle$ the vacuum expectation value (vev) of the Higgs doublet Φ. Diagonalizing M_R brings us to the physical basis

$$U_R^T\, M_R\, U_R = \mathrm{diag}(M_1, M_2, M_3) \tag{2.5}$$

and defines the physical states

$$N_R = U_R^\dagger\, N'_R\,. \tag{2.6}$$

In the new basis the Dirac mass matrix appearing in the first part of the lagrangian eq. (2.4) also changes to

$$m_D = \tilde{m}_D\, U_R\,. \tag{2.7}$$

It is the rotated Dirac mass matrix which determines the lepton asymmetry. The asymmetry is caused by the interference of tree level with one-loop corrections for the decays of the lightest Majorana, $N_1 \to \Phi\, l^c$ and $N_1 \to \Phi^\dagger\, l$:

$$\varepsilon = \frac{\Gamma(N_1 \to \Phi\, l^c) - \Gamma(N_1 \to \Phi^\dagger\, l)}{\Gamma(N_1 \to \Phi\, l^c) + \Gamma(N_1 \to \Phi^\dagger\, l)} = \frac{1}{8\,\pi\, v^2} \frac{1}{(m_D^\dagger m_D)_{11}} \sum_{j=2,3} \mathrm{Im}(m_D^\dagger m_D)^2_{1j}\, f\left(\frac{M_j^2}{M_1^2}\right). \tag{2.8}$$

Here, $v \simeq 174$ GeV is the weak scale and the function f includes terms from vertex [3, 18] and self-energy [19] contributions:

$$f(x) = \sqrt{x}\left(1 + \frac{1}{1-x} - (1+x)\,\ln\left(\frac{1+x}{x}\right)\right) \simeq -\frac{3}{2\,\sqrt{x}}\,. \tag{2.9}$$

The approximation holds for $x \gg 1$. To calculate the asymmetry ε, we need the Dirac mass matrix $\tilde{m}_D$ and the matrix U_R in order to obtain m_D in eq. (2.7). In general, after introducing $\tilde{m}_D$ in the Lagrangian, one may obtain a very different m_D after the rotation with U_R.

In our approach, the left-right symmetry [20] plays an important role. It relates the unitary matrices U_L and U_R to each other since the triplet induced Majorana mass matrices in eq. (2.1) have the same coupling matrix f in generation space:

$$m_L = f\, v_L \quad \text{and} \quad M_R = f\, v_R\,. \tag{2.10}$$

The numbers $v_{L,R}$ are the vacuum expectation values (vevs) of the left- and right-handed Higgs triplets, whose existence is needed to maintain the left-right symmetry. They receive their vevs at the minimum of the potential, producing at the same time masses for the gauge bosons. In general [20], this results in

$$v_L\, v_R \simeq \gamma\, M_W^2\,, \tag{2.11}$$

where the constant γ is a model dependent parameter of $\mathcal{O}(1)$. Inserting this equation as well as eq. (2.10) in (2.2) yields

$$m_\nu = v_L\, \left(f - \tilde{m}_D\, \frac{f^{-1}}{\gamma M_W^2}\, \tilde{m}_D^T \right) . \tag{2.12}$$

As stated before, the contribution of the left- handed Higgs triplet to m_ν is often neglected. However, if one compares the relative magnitude of the two contributions in eq. (2.2), denoting the largest mass in the Dirac matrix with m, one finds

$$\frac{|\tilde{m}_D\, M_R^{-1}\, \tilde{m}_D^T|}{|m_L|} \simeq \frac{m^2/v_R}{v_L} \simeq \frac{m^2}{\gamma\, M_W^2}\,. \tag{2.13}$$

Here, we only used eq. (2.11) and assumed that the matrix elements of f and f^{-1} are of the same order of magnitude. It is seen that this ratio is of order one only for the top quark mass, i.e. if one identifies the Dirac mass matrix with the up quark mass matrix. Due to the hierarchical structure of quark masses, the up quark mass matrix can be written as

$$\tilde{m}_D \simeq \mathrm{diag}(0, 0, m_t)\,, \tag{2.14}$$

where m_t denotes the top quark mass. This means that only the (33) entry of m_ν has a contribution from $\tilde{m}_D\, M_R^{-1}\, \tilde{m}_D^T$. One can then solve for this term and with the experimental knowledge of m_ν obtain f. With f and eq. (2.10) we have M_R, whose diagonalization gives U_R, which in turn gives m_D through eqs. (2.7,2.14). Then, via eq. (2.8), one obtains the lepton asymmetry. This program has been performed in [13], finding that from the solutions to the solar neutrino problem only the small angle MSW and vacuum oscillations give an asymmetry consistent with the experimental bound (1.1). The large angle MSW solution gives a very large asymmetry and must be suppressed by fine-tuning the phases in the mixing matrix. The identification of $\tilde{m}_D$ with the up quark mass matrix follows in simple SO(10) models with a 10-plet of Higgses generating the fermion masses. The mass relation is different in more general situations, when the Higgses belong to the 126 representation [21]. There are also examples where the Dirac masses for neutrinos are zero at tree level and are generated by radiative corrections [22]. In these cases the scale of the Dirac mass is much smaller.

If we now identify $\tilde{m}_D$ with the down quark or charged lepton mass matrix, then the ratio in eq. (2.13) is always much smaller than one, so that the second term in eq. (2.12) can be neglected and it follows

$$f \simeq \frac{1}{v_L}\, m_\nu\,. \tag{2.15}$$

Therefore, with the help of eqs. (2.3,2.5,2.10), one arrives at a very simple connection between the left- and right-handed neutrino sectors:

$$U_R = U_L \text{ and } M_i = m_i\, \frac{v_R}{v_L}\,. \tag{2.16}$$

The striking property is that the light neutrino masses are proportional to the heavy ones. In addition, the left- and right-handed rotation is the same. This is the main result of this work and will be used in the following analysis.[1]

We finally specify the order of magnitude of $v_{L,R}$. The maximal size of $m_\nu = v_L\, f$ is $\sqrt{\Delta m_A^2} \lesssim 0.1\,\mathrm{eV}$, which is only compatible with $v_L\, v_R \simeq \gamma\, M_W^2$ for $v_L \simeq 0.1\,\mathrm{eV}$ and $v_R \simeq 10^{15}\,\mathrm{GeV}$, as long as $f \simeq 0.1 \ldots 1$. This means that v_R is close to the grand unification scale and v_L is of order of the neutrino masses, which is expected since m_L is the only contribution to m_ν.

3. Estimates

3.1 Neutrino mass matrix

The matrix U_L is measured in neutrino oscillation experiments, a convenient parametrisation is

$$\begin{aligned} U_L &= U_{\mathrm{CKM}}\ \mathrm{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)}) \\ &= \begin{pmatrix} c_1 c_3 & s_1 c_3 & s_3 e^{-i\delta} \\ -s_1 c_2 - c_1 s_2 s_3 e^{i\delta} & c_1 c_2 - s_1 s_2 s_3 e^{i\delta} & s_2 c_3 \\ s_1 s_2 - c_1 c_2 s_3 e^{i\delta} & -c_1 s_2 - s_1 c_2 s_3 e^{i\delta} & c_2 c_3 \end{pmatrix} \mathrm{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)})\,, \end{aligned} \tag{3.1}$$

where $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$. This matrix connects gauge eigenstates ν_α ($\alpha = e, \mu, \tau$) with mass eigenstates ν_i ($i = 1, 2, 3$), i.e.

$$\nu_\alpha = U_{L\alpha i}\, \nu_i.$$

Note that we identify the neutrino mixing matrix in eq. (3.1) with the matrix U_L diagonalizing the neutrino mass matrix eq. (2.3). This assumes implicitly that the charged lepton mixing is small.

The "CKM-phase" δ may be probed in oscillation experiments, as long as the large mixing angle solution is the solution to solar oscillations [23]. The other two "Majorana phases" α and β can be investigated in neutrinoless double beta decay [24,

[1] In the language of [13] one can say that the work in this paper represents the special case $s = 0$ and $m_t \to m$.

25]. The choice of the parameterization in eq. (3.1) reflects this fact since the *ee* element of the mass matrix $\sum_i U_{Lei}^2 m_i$ depends only on the phases α and β. In a hierarchical scheme, to which we will limit ourselves, there is no constraint on the phases from neutrinoless double beta decay [25]. Thus, we can choose them arbitrarily. The mass eigenstates are given as

$$\begin{aligned} m_3 &= \sqrt{\Delta m_A^2 + m_2^2} \simeq \sqrt{\Delta m_A^2} \\ m_2 &= \sqrt{\Delta m_\odot^2 + m_1^2} \simeq \sqrt{\Delta m_\odot^2} \gg m_1 \,. \end{aligned} \tag{3.2}$$

The values of θ_2 and Δm_A^2 are known to a good precision, corresponding to maximal mixing $\theta_2 \simeq \pi/4$ and $\Delta m_A^2 \simeq 3 \times 10^{-3}\,\mathrm{eV}^2$. Regarding θ_1 and $\Delta m_\odot^2$ three distinct areas in the parameter space are allowed, small (large) mixing, denoted by SMA (LMA) and quasi-vacuum oscillations (QVO):

$$\begin{aligned} &\text{SMA:} \quad \tan^2\theta_1 \simeq 10^{-4}\ldots 10^{-3}, \quad \Delta m_\odot^2 \simeq 10^{-6}\ldots 10^{-5}\,\mathrm{eV}^2 \\ &\text{LMA:} \quad \tan^2\theta_1 \simeq 0.1\ldots 4, \quad \Delta m_\odot^2 \simeq 10^{-5}\ldots 10^{-3}\,\mathrm{eV}^2 \;. \\ &\text{QVO:} \quad \tan^2\theta_1 \simeq 0.2\ldots 4, \quad \Delta m_\odot^2 \simeq 10^{-10}\ldots 10^{-7}\,\mathrm{eV}^2 \end{aligned} \tag{3.3}$$

For the last angle θ_3 there exists only a limit of about $\sin^2\theta_3 \lesssim 0.08$. See [26] for a recent three-flavor fit to all available data and [27] for a more general discussion of the derivation of neutrino mass matrices.

The results for atmospheric and solar mixing imply two simple forms for U_L, namely

$$U_L \simeq \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & s_3 e^{-i\delta} \\ -\frac{1}{2}(1+s_3 e^{i\delta}) & \frac{1}{2}(1-s_3 e^{i\delta}) & \frac{1}{\sqrt{2}} \\ \frac{1}{2}(1-s_3 e^{i\delta}) & -\frac{1}{2}(1+s_3 e^{i\delta}) & \frac{1}{\sqrt{2}} \end{pmatrix} \mathrm{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)}) \tag{3.4}$$

for LMA as well as QVO and

$$U_L \simeq \begin{pmatrix} 1 & 0 & s_3 e^{-i\delta} \\ -\frac{1}{2}s_3 e^{i\delta} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2}s_3 e^{i\delta} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \mathrm{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)}) \tag{3.5}$$

for the SMA case. These forms can be used to estimate the magnitude of the baryon asymmetry within our scenario.

3.2 Out-of-equilibrium condition

Besides CP and lepton-number violation, a necessary condition [28] for leptogenesis to work is the out-of-equilibrium decay of the heavy Majorana neutrinos, i.e.

$$K \equiv \frac{\Gamma_1}{H(T=M_1)} = \frac{(m_D^\dagger m_D)_{11}\, M_1}{8\,\pi\, v^2} \frac{M_{\mathrm{Pl}}}{1.66\,\sqrt{g^*}\, M_1^2} \leq 1\,, \tag{3.6}$$

where Γ_1 is the width of the lightest Majorana neutrino and $H(T = M_1)$ the Hubble constant at the temperature of the decay. The number of massless degrees of freedom at the time of the decay is $g^* \simeq 110$ and $M_{\rm Pl}$ is the Planck mass. However, values of K smaller than ten still give sizable lepton asymmetry, even though the asymmetry is reduced by lepton-number violating wash-out processes. This suppression is obtained by integrating the Boltzmann equations and can be parameterized by [29]

$$\kappa \simeq \left\{ \begin{array}{ccc} \sqrt{0.1\,K}\,\exp(-4/3\,(0.1\,K)^{0.25}) & \text{for} & K \gtrsim 10^6 \\ \dfrac{0.3}{K\,(\ln K)^{0.6}} & \text{for} & 10 \lesssim K \lesssim 10^6 \\ \dfrac{1}{2\,\sqrt{K^2\,+9}} & \text{for} & 0 \lesssim K \lesssim 10 \end{array} \right. . \tag{3.7}$$

Since also the down quarks and charged leptons display a mass hierarchy, we may write the Dirac mass matrix in analogy to eq. (2.14) as

$$\tilde{m}_D \simeq \mathrm{diag}(0,0,m)\,, \tag{3.8}$$

where m might be the bottom quark or tau lepton mass. Inserting this in eq. (2.7) and using the approximate forms for U_L in eqs. (3.4,3.5) yields

$$(m_D^\dagger m_D)_{11} = |U_{L\tau 1}|^2\,m^2 \simeq \frac{m^2}{4} \left\{ \begin{array}{cc} (1 - 2\,s_3\,c_\delta) & \text{for LMA, QVO} \\ s_3^2 & \text{for SMA} \end{array} \right. , \tag{3.9}$$

where we took only the leading term in s_3 and set $c_3 = 1$. Using eq. (2.16) we replace M_1 by $\frac{v_R}{v_L}\,m_1$ and all other quantities by known constants to arrive at

$$K \simeq 1.5 \left(\frac{10^{-6}\,\mathrm{eV}}{m_1}\right) \left(\frac{10^{15}\,\mathrm{GeV}}{v_R}\right)^2 \gamma \left(\frac{m}{\mathrm{GeV}}\right)^2 \left\{ \begin{array}{cc} (1 - 2\,s_3\,c_\delta) & \text{for LMA, QVO} \\ s_3^2 & \text{for SMA} \end{array} \right. . \tag{3.10}$$

Thus, for typical parameters, the dilution factor is not too small. We shall assume for the following estimates the value $\kappa \lesssim 1/6$.

3.3 Lepton and baryon asymmetry

From eqs. (2.7, 3.8) and the approximate forms of U_L we calculate the imaginary parts of $(m_D^\dagger m_D)_{1j}$ needed for calculating the lepton asymmetry in eq. (2.8). For the 12 element

$$\mathrm{Im}(m_D^\dagger m_D)_{12}^2 = m^4\,\mathrm{Im}(U_{L\tau 1}^* U_{L\tau 2})^2 \tag{3.11}$$

$$\simeq \frac{m^4}{16} \left\{ \begin{array}{cc} (s_{2\alpha} + 4\,s_3\,s_\delta\,c_{2\alpha}) & \text{for LMA, QVO} \\ 2\,s_3^2\,s_{2(\alpha-\delta)} & \text{for SMA} \end{array} \right. \tag{3.12}$$

and for the 13 element

$$\mathrm{Im}(m_D^\dagger m_D)_{13}^2 = m^4\,\mathrm{Im}(U_{L\tau 1}^* U_{L\tau 3})^2 \tag{3.13}$$

$$\simeq \frac{m^4}{8}\begin{cases} (s_{2(\beta+\delta)} - 2\,s_3\,s_{2\beta+\delta}) & \text{for LMA, QVO} \\ s_3^2\,s_{2\beta} & \text{for SMA} \end{cases} \tag{3.14}$$

with the notation $s_{2(\alpha-\delta)} = \sin 2(\alpha-\delta)$ and so on. Note that the asymmetry ε is proportional to $(m_D^\dagger m_D)_{11}^{-1}$ and therefore to s_3^{-2} for the SMA case. This might enhance the lepton asymmetry dramatically. However, as seen from the last two equations, for the SMA case, $\mathrm{Im}(m_D^\dagger m_D)_{1j}^2$ is proportional to s_3^2, which cancels this potentially dangerous term.

From the lepton asymmetry ε the baryon asymmetry Y_B is obtained by

$$Y_B = c\,\kappa\frac{\varepsilon}{g^*}\,, \tag{3.15}$$

where $c \simeq -0.55$, denoting the fraction of the lepton asymmetry converted into a baryon asymmetry via sphaleron [30] processes. Using the approximate form of the function f in eq. (2.9), the mass eigenstates from eq. (3.2) and the fact that $M_i/M_j = m_i/m_j$ one finally finds for the baryon asymmetry for the LMA and QVO case

$$Y_B \cdot 10^{10} \lesssim 4.1\,\left(\frac{m}{\mathrm{GeV}}\right)^2 \frac{1}{1-2\,s_3\,c_\delta}\left\{(s_{2\alpha} + 4\,s_3\,s_\delta\,c_{2\alpha})\frac{m_1}{\sqrt{\Delta m_\odot^2}} + \right. \\ \left. + 2(s_{2(\beta+\delta)} - 2\,s_3\,s_{2\beta+\delta})\frac{m_1}{\sqrt{\Delta m_A^2}}\right\} \tag{3.16}$$

whereas in the SMA case one gets

$$Y_B \cdot 10^{10} \lesssim 8.2\,\left(\frac{m}{\mathrm{GeV}}\right)^2\left\{s_{2(\alpha-\delta)}\frac{m_1}{\sqrt{\Delta m_\odot^2}} + s_{2\beta}\frac{m_1}{\sqrt{\Delta m_A^2}}\right\}. \tag{3.17}$$

Thus, for comparable values of the phases and $\Delta m_\odot^2 \ll \Delta m_A^2$ the first term dominates Y_B. Note that Y_B vanishes for $\alpha = \beta = \delta = 0$, in which CP is conserved, and that it is proportional to the square of the largest mass in the Dirac mass matrix. Therefore, if one would take a charged lepton mass matrix, the results for Y_B will change roughly by a factor of $m_\tau^2/m_b^2 \simeq 0.2$.

To sum up, identifying the Dirac mass matrix with the down quark or charged lepton mass matrix we obtain the simple relations in eq. (2.16) which introduces the lightest left-handed neutrino mass in the answers. In our previous paper [13] the presence of the top quark mass did not permit such a simplification because a different diagonalization for U_L and U_R had to be performed. As a consequence, M_1 was not directly proportional to m_1 but received contributions from m_2, m_3 and a term of order $m_t\,M_W/v_R$. These contributions are all larger than m_1 so that basically no dependence on m_1 was evident.

3.4 Limit on smallest mass eigenstate

The approximate form of Y_B in eqs. (3.16), (3.17) can be used to derive a limit on the smallest mass eigenstate m_1. In principle, it can be exactly zero, since oscillation experiments only measure the difference of the square of two masses. As mentioned already, in [13] there was practically no dependence on m_1 whereas here it is decisive since Y_B is directly proportional to m_1. To get the highest possible Y_B we choose the phases such that their contribution is maximal and take the lowest possible $\Delta m_\odot^2$, as listed in eq. (3.3). The factor $(1-2\,s_3\,c_\delta)^{-1}$ can give for $s_3 \lesssim 0.3$ give an enhancement of about 2.5 at most so that for $m = m_b$ we obtain:

$$Y_B \cdot 10^{10} \lesssim \left\{ \begin{array}{ll} \left(\frac{m_1}{10^{-5}\,\mathrm{eV}}\right) \cdot 0.6 & \text{for SMA} \\ \left(\frac{m_1}{10^{-6}\,\mathrm{eV}}\right) \cdot 1.4 & \text{for LMA} \\ \left(\frac{m_1}{10^{-7}\,\mathrm{eV}}\right) \cdot 2.1 & \text{for QVO} \end{array} \right. \tag{3.18}$$

Therefore, to obtain an asymmetry within the experimental range, m_1 should not be much smaller than 10^{-6} (10^{-7}, 10^{-8}) eV for the SMA (LMA, QVO) case.

4. Numerical results

To check the estimates made in the previous section we show in figure 1 the baryon asymmetry for three typical values of the three solar solutions,

$$\begin{array}{lll} \Delta m_\odot^2 = 5 \cdot 10^{-6}\,\mathrm{eV}^2\,, & \tan^2\theta_1 = 5 \cdot 10^{-4} & \text{SMA} \\ \Delta m_\odot^2 = 5 \cdot 10^{-5}\,\mathrm{eV}^2\,, & \tan^2\theta_1 = 1 & \text{LMA} \\ \Delta m_\odot^2 = 10^{-8}\,\mathrm{eV}^2\,, & \tan^2\theta_1 = 1 & \text{QVO} \end{array} \tag{4.1}$$

fixed $\Delta m_A^2 = 3 \cdot 10^{-3}$ eV^2, $\theta_2 = \pi/4$ and choose the phases $3\,\alpha = 5\,\beta = 6\,\delta = \pi$. The two parameters of the left-right symmetry, v_R and γ were taken 10^{15} GeV and 1, respectively. For $\tilde{m}_D$ we took a typical down quark mass matrix (see e.g. [31])

$$\tilde{m}_D = \begin{pmatrix} 0 & \sqrt{m_d\,m_s} & 0 \\ \sqrt{m_d\,m_s} & m_s & \sqrt{m_b\,m_s} \\ 0 & \sqrt{m_b\,m_s} & m_b \end{pmatrix}, \tag{4.2}$$

where $m_{d,s,b}$ are the masses of the d, s and b quark. The lowest mass eigenstate m_1 is chosen 10^{-5} $(2 \cdot 10^{-6})$ eV for the SMA and LMA (QVO) case. We find that there is practically no difference when one takes this matrix or the simple form in eq. (3.8), showing the model independence of our scenario.

For the SMA case there are areas, in which Y_B becomes very small and even negative. They are not found by our estimates and can be shown to stem from cancellations of terms proportional to $\sin^2\theta_3$ and $\sin^2\theta_1$. There is no significant dependence on $\sin^2\theta_3$ for the LMA case, as also indicated by the approximate expressions

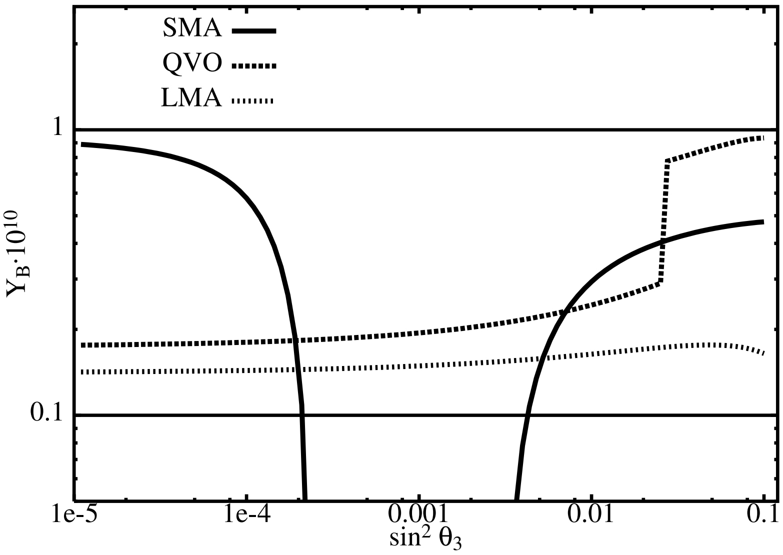

Figure 1: Behavior of the baryon asymmetry as a function of $\sin^2 \theta_3$ for the parameters as given in section 4. For this plot, the Dirac mass matrix is a down quark mass matrix.

in eqs. (3.16), (3.17). Regarding QVO, one sees that for larger s_3 the dilution factor rises from about 0.01 to its maximal value. This explains the rise of Y_B with s_3^2 for this solution. With comparable phases, the asymmetry of the LMA and SMA case can be of the same order whereas the one for QVO needs suppression through m_1 due to the very low $\Delta m_\odot^2$. If one inserts our chosen parameters in the approximate forms for K and Y_B in eqs. (3.10), (3.16), (3.17), then one finds for $s_3 = 0$ that

$$Y_B \cdot 10^{10} \simeq \begin{cases} 0.7 & \text{for SMA} \\ 0.1 & \text{for LMA} \\ 0.1 & \text{for QVO} \end{cases} \tag{4.3}$$

which is in good agreement with the exact result.

5. Connection to Neutrinoless Double Beta Decay

As mentioned before, only the LMA solution provides the possibility to find leptonic CP violation in oscillation experiments [23]. In addition, in the hierarchical mass scheme only the LMA solution might produce a measurable Majorana mass for the electron neutrino [32]. Fortunately, the latest SuperKamiokande data [33] seems to indicate that LMA is the preferred solution of the solar neutrino problem. The Majorana mass of the electron neutrino is defined as

$$\langle m \rangle = \sum_i U_{Lei}^2 \, m_i \tag{5.1}$$

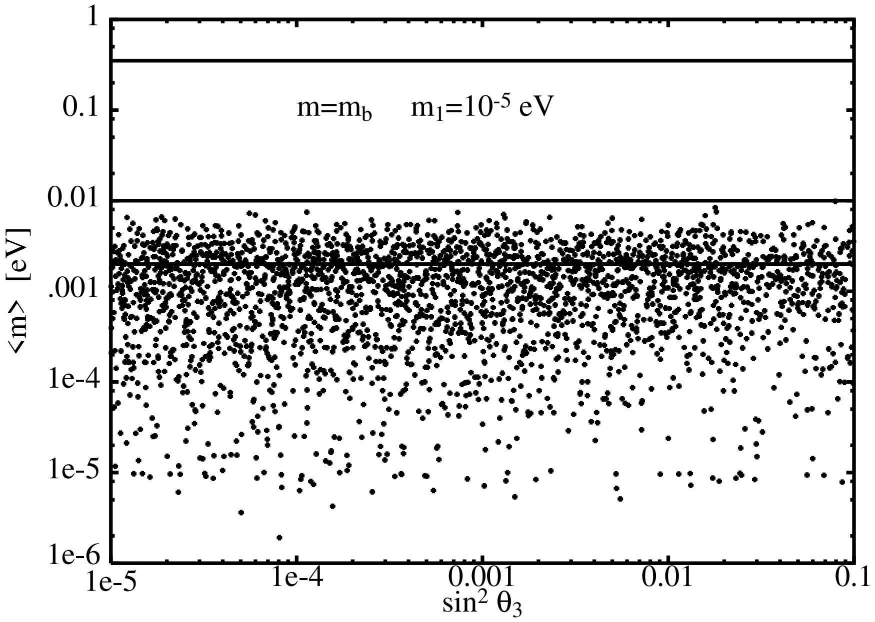

Figure 2: The effective electron neutrino mass $\langle m\rangle$ as a function of $\sin^2\theta_3$ for $m_1 = 10^{-5}$ eV. 10000 random points were generated, about 2700 give Y_B within 0.1 and 1 and less than 800 an effective mass $\langle m\rangle$ above 0.002 eV. For this plot, the Dirac mass matrix is a down quark mass matrix.

and due to the complex matrix elements $U_{L\alpha i}$ there is a possibility of cancellation [25] of terms in eq. (5.1). It is therefore interesting to ask if the parameters that produce a satisfying Y_B also deliver a sizable $\langle m\rangle$. In figure 2 we show the results of a random scan of the parameter space with a down quark Dirac mass matrix, taking the following values: $\Delta m^2_\odot$ between 10^{-5} and 10^{-3} eV^2, $\tan^2\theta_1$ between 0.1 and 4 and the three phases between zero and 2π. The atmospheric parameters were fixed as before. We choose for the lowest mass eigenstate $m_1 = 10^{-5}$ eV. Included in the plot is the current limit of $\langle m\rangle \lesssim 0.35$ eV [34] and two future limits, 0.01 and 0.002 eV. From 10000 parameter sets, about 2700 give a baryon asymmetry of the correct magnitude. However, most of the points (about 2000) lie below the lowest achievable $\langle m\rangle$ limit of 0.002 eV. For higher m_1 the fraction of parameter sets giving an acceptable Y_B decreases. For $m_1 = 10^{-3}$ eV, only 5 % give acceptable Y_B and $\langle m\rangle$.

If on the other hand we identify the Dirac mass matrix with a charged lepton mass matrix, the asymmetry is reduced. A charged lepton mass matrix is obtained from eq. (4.2) by replacing the bottom (charm, down) quark mass with the tau (muon, electron) mass. Then, the asymmetry needs a smaller suppression through the phases and $\langle m\rangle$ can be bigger, going up to its maximal value of about $\frac{1}{2}\sqrt{\Delta m^2_{\odot\max}} \simeq 0.016$ eV. This can be seen in figure 3 for $m_1 = 10^{-3}$ eV. Here, about 35 % of the parameter sets give a correct Y_B and 29 % an $\langle m\rangle$ above 0.002 eV. For lower m_1, this fraction decreases rapidly because the asymmetry decreases.

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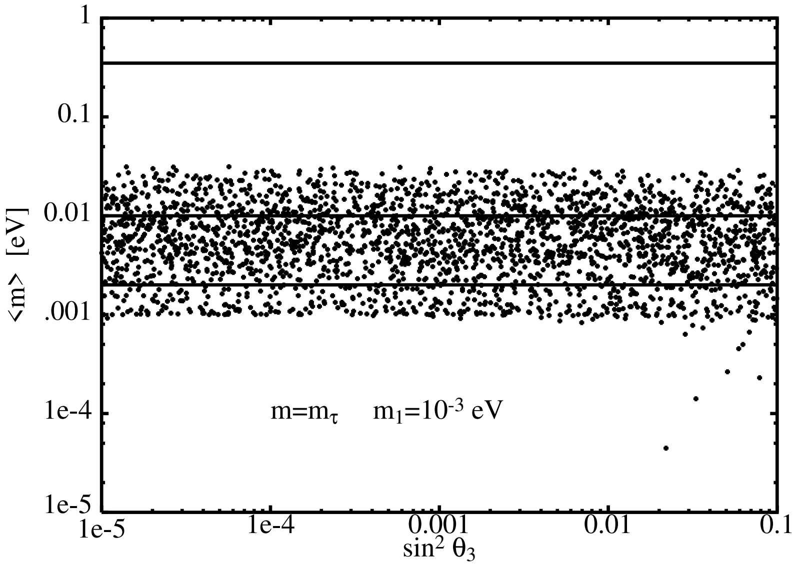

Figure 3: The effective electron neutrino mass $\langle m \rangle$ as a function of $\sin^2\theta_3$ for $m_1 = 10^{-3}$ eV. 7000 random points were generated, about 2500 give Y_B within 0.1 and 1 and about 2100 an effective mass $\langle m \rangle$ above 0.002 eV. For this plot, the Dirac mass matrix is a charged lepton mass matrix.

The "CKM-phase" δ does not appear in $\langle m \rangle$. It appears in the asymmetry but turns out to be not very strongly restricted. The reason is that in our parametrisation (3.1) the phase δ is often connected with the small quantity s_3, which results in a small overall dependence of the asymmetry on this phase. Therefore, one can expect a sizable $\langle m \rangle$ within our scenario as long as the smallest mass eigenstate m_1 is not much lighter than 10^{-3} eV and the Dirac mass matrix is identified with the charged lepton mass matrix. The possibility of detecting CP violating effects in oscillation experiments remains open, in the sense that within our scenario the phase δ can take any value. Finally, we remark that for the scenario in [13] an extreme fine-tuning of the parameters is required to obtain a reasonable Y_B. Therefore, no sizable signal in neutrinoless double beta decay can be expected if one identifies the Dirac mass matrix with the up quark mass matrix.

6. Conclusions

In a left-right symmetric theory with a left-handed Higgs triplet the light neutrino mass matrix originates from its vev. If in addition the Dirac mass matrix in the see-saw formula is identified with the down quark or charged lepton mass matrix, then a simple connection between the light left-handed and heavy right-handed neutrino sectors emerges. The out-of-equilibrium condition for the decay of the heavy Majorana neutrinos is automatically fulfilled and a baryon asymmetry of the correct order of magnitude is produced. The limit on the smallest mass eigenstate m_1

is approximately 10^{-6} (10^{-7}, 10^{-8}) for the SMA (LMA, QVO) cases. Within this scenario a sizable effect in neutrinoless double beta decay is expected, as long as m_1 is not lighter than 10^{-3} eV and the Dirac mass matrix is the charged lepton mass matrix. The possibility of finding CP violation in oscillation experiments remains open.

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